Introduction to Set Theory

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Set?

A set is a collection of well-defined and different elements. A set can be written explicitly by listing its elements using curly bracket. A set is typically determined by its distinct elements, or members, by which we mean that the order does not matter, and if an element is repeated several times, we only care about one instance of the element. We typically use the bracket notation { } to refer to a set.
Example: The sets {1, 2, 3} and {3, 1, 2} are the same, because the ordering does not matter. The set {1, 1, 1, 2, 3, 3, 3} is also the same set as {1, 2, 3}, because we are not interested in repetition: either an element is in the set, or it is not, but we do not count how many times it appears.

Representation of a Set:

Sets can be represented in two ways:

- <u>Roster or Tabular Form or Explicit Form</u>

The set is represented by listing all the elements comprising it. The elements are enclosed within braces and separated by commas.
Example: Set of vowels in English alphabet, A = {a, e, i, o, u}

- <u>Set Builder Notation or Implicit Form</u>

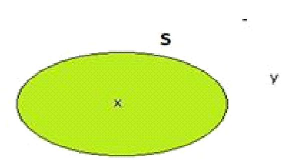
The set is defined by specifying a property that elements of the set have in common. The set is described as A = {x: p(x)}.
Example: The set {a, e, i, o, u} is written as:
A = {x: x is a vowel in English alphabet}

Cardinality of a Set (Size of the Set)

Cardinality of a set S, denoted by $|S|$, is the number of distinct elements of the set. The number is also referred as the cardinal number. If a set has an infinite number of elements, its Cardinality is ∞.
Example: $|\{1, 4, 3, 5\}| = 4$, $|\{1, 2, 3,4,5,\ldots\}| = \infty$

Membership

If an element x is a member of any set S, it is denoted by $x \in S$ and if an element y is not a member of set S, it is denoted by $y \notin S$.
Example: If S = {1, 1.2, 1.7, 2}, $1 \in S$ but $1.5 \notin S$



$x \in S$ and $y \notin S$

Subset

A set X is a subset of set Y (Written as $X \subseteq Y$) if every element of X is an element of set Y.
Example 1: Let, X = {1, 2, 3, 4, 5, 6 } and Y = { 1, 2 }. Here set Y is a subset of set X as all the elements of set Y is in set X. Hence, we can write $Y \subseteq X$.

Proper Subset:

The term "proper subset" can be defined as "subset of but not equal to". A Set X is a proper subset of set Y (Written as $X \subset Y$) if every element of X is an element of set Y and $|X| < |Y|$.
Example: Let, X = {1, 2, 3, 4, 5, 6} and Y = {1, 2}. Here set $Y \subset X$ since all elements in Y are contained in X too and X has at least one element is more than set Y.

Note:

- Every set is a subset of itself.
- The empty set is a subset of every set.
- The total number of subsets of a finite set containing n elements is 2n.

Standard Notation of sets:

N = {1, 2, . . .}, the set of Natural numbers;

W = {0, 1, 2, . . .}, the set of whole numbers

Z = {0, 1, −1, 2, −2, . . .}, the set of Integers;

Q = { p q : p, q ∈ Z, q ≠ 0}, the set of Rational numbers;

R = the set of Real numbers; and

C = the set of Complex numbers.

Types of Sets:

Sets can be classified into many types. Some of which are finite, infinite, universal, singleton set, etc.

- **Finite Set**

A set which contains a definite number of elements is called a finite set.
Example: S = {x | x ∈ N and 70 > x > 50}

- **Infinite Set**

A set which contains infinite number of elements is called an infinite set.
Example: S = {x | x ∈ N and x > 10}

- **Universal Set**

It is a collection of all elements in a particular context or application. All the sets in that context or application are essentially subsets of this universal set. Universal sets are represented as U.
Example: We may define U as the set of all animals on earth. In this case, set of all mammals is a subset of U, set of all fishes is a subset of U, set of all insects is a subset of U, and so on.

- **Empty Set or Null Set**

An empty set contains no elements. It is denoted by ∅. As the number of elements in an empty set is finite, empty set is a finite set.The Cardinality of empty set or null set is zero.
Example: S = {x | x ∈ N and 7 < x < 8} =∅

- **Singleton Set or Unit Set**

Singleton set or unit set contains only one element. A singleton set is denoted by {s}.
Example: S = {x | x ∈ N, 7 < x < 9} = { 8 }

- **Equal Set**

If two sets contain the same elements they are said to be equal.
Example: If A = {1, 2, 6} and B = {6, 1, 2}, they are equal as every element of set A is an element of set B and every element of set B is an element of set A.

- **Equivalent Set**

If the cardinalities of two sets are same, they are called equivalent sets.
Example: If A = {1, 2, 6} and B = {16, 17, 22}, they are equivalent as cardinality of A is equal to the cardinality of B. i.e. |A|=|B|=3

- **Disjoint Set**

Two sets A and B are called disjoint sets if they do not have even one element in common.

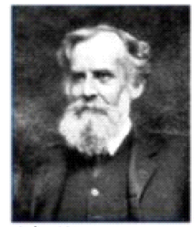
- **Overlapping Set:**

Two sets that have at least one common element are called overlapping sets.
Example: Let, A = {1, 2, 6} and B = {6, 12, 42}.There is a common element '6'; hence these sets are overlapping sets.

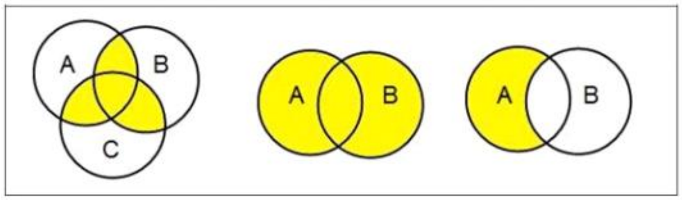
Venn Diagrams

Venn diagram, invented in 1880 by John Venn, is a schematic diagram that shows all possible logical relations between different mathematical sets.

In Venn-diagrams the universal set U is represented by point within a rectangle and its subsets are represented by points in closed curves (usually circles) within the rectangle.



John Venn
(1834-1923)



Examples of Venn-diagrams

Set Operations:

Set Operations include Set Union, Set Intersection, Set Difference, Complement of Set, and Cartesian product.

- **Set Union**

The union of sets A and B (denoted by A ∪ B) is the set of elements which are in A, in B, or in both A and B. Hence, AUB = {x | x ∈A or x ∈B}.

Example: If A = {10, 11, 12, 13} and B = {13, 14, 15}, then AUB = {10, 11, 12, 13,14, 15}. (The common element occurs only once)

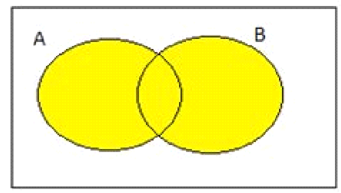


Figure: Venn diagram of AUB

- **Set Intersection**

The intersection of sets A and B (denoted by A ∩ B) is the set of elements which are in both A and B. Hence, A∩B = {x | x ∈A and x ∈B}.

Example: If A = {11, 12, 13} and B = {13, 14, 15}, then A∩B = {13}.

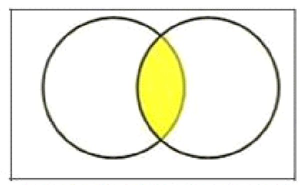


Figure: Venn diagram of A∩B

- **Disjoint Sets**

Two sets A and B are called disjoint sets if they do not have even one element in common i.e. n (A ∩ B) = ∅

Example: If A ={2,3} and B ={5,6} Here A and B do not have any common element.

- **Complement of a Set**

The complement of a set A (denoted by A’) is the set of elements which are not in set A. Hence, A' = {x | x ∉A}.

More specifically, A'= (U–A) where U is a universal set which contains all objects.

Example: If A ={x | x belongs to set of odd integers} then A' ={y | y does not belong to set of odd integers}

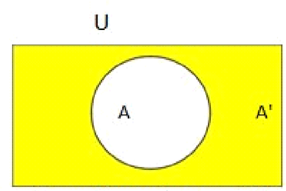


Figure: Venn diagram of A'

- **Symmetric difference of two sets**

For any set A and B, their symmetric difference (A – B) ∪ (B – A) is defined as set of elements which do not belong to both A and B. It is denoted by A ∆ B.

Hence, A ∆ B = (A – B) ∪ (B – A) = {x : x ∉ A ∩ B}.

Example: If A = {10, 11, 12, 13} and B = {13, 14, 15}, then (A—B) = {10, 11, 12} and (B—A) = {14,15}.

A ∆ B = (A – B) ∪ (B – A)={10,11,12,14,15}.

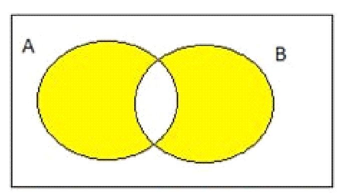


Figure: Venn diagram of A ∆ B = (A – B) ∪ (B – A)

- **Set Difference/ Relative Complement**

The set difference of sets A and B (denoted by A–B) is the set of elements which are only in A but not in B. Hence, A—B = {x | x ∈A and x ∉B}.

Example: If A = {10, 11, 12, 13} and B = {13, 14, 15}, then (A—B) = {10, 11, 12} and (B—A) = {14,15}. Here, we can see (A—B) ≠ (B—A).

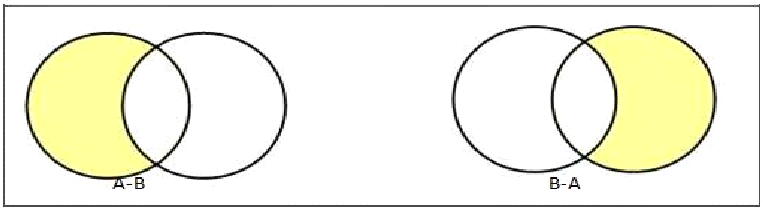


Figure: Venn diagram of A–B and B–A.

Some Properties of Complement of Sets

$A \cup A' = U$

$A \cap A' = \Phi$

$U' = \Phi$

$\Phi' = U$

$(A')' = A$

Why are Sets Important?

Sets are fundamental property of mathematics. Now as a word of warning, sets, by themselves, seem pretty pointless. But it's only when we apply sets in different situations do they become the powerful building block of mathematics that they are. Math can get amazingly complicated quite fast. Graph Theory, Abstract Algebra, Real Analysis, Complex Analysis, Linear Algebra and the list goes on. But there is one thing that all of these share in common i.e. Sets
Set Theory is an important tool for formalizing and reasoning about comparison. Knowledge of the concept helps to think abstractly.

Set Theory Origin

Set Theory is a branch of mathematics that deals with the properties of well-defined collections of objects, which may or may not be of a mathematical nature, such as numbers or functions.
The theory of sets was developed by German mathematician, Georg Cantor. He first encountered sets while working on "Problems on Trigonometric Series". He had defined a set as a collection of definite and distinguishable objects selected by the means of certain rules or description.

Applications of Mathematics for CSE

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- Graphics and Animations